Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Partial Differential Equations

Subject Code: 5SC02PDE1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 25/04/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Find the characteristic of $(\sin^2 x)r + (2\cos x)s - t = 0$. **(02)**

b. Find order of equation $(5D' - 2)(3D' + D^2)^2 z = 0$. **(01)**

c. Classify the region in which equation **(01)**

$x(xy-1)r - (x^2y^2-1)s + y(xy-1)t + xp + yq = 0$ is parabolic.

d. Find particular integral of $(DD' + aD + bD' + ab)z = e^{mx+ny}$. **(01)**

e. The equation $\frac{\partial^2 u}{\partial x \partial y} + 3x\left(\frac{\partial z}{\partial x}\right)^2 - xy\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) + y^2\left(\frac{\partial^2 z}{\partial y^2}\right) = 0$ is same as **(01)**

$s + 3xp^2 - xypq + y^2 t = 0$. Determine whether the statement is True or False.

f. The equation $x^2 r - 2xypq + y^2 t = 0$ is nonlinear. Determine whether the statement is True or False. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Reduce $yr + (x+y)s + xt = 0$ to canonical form and find its solution. **(07)**

b. Using Monge's method, solve the equation $3s - 2(rt - s^2) = 2$. **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a. Using Monge's method, solve the equation $r + 4s + 3t = xy$. **(07)**

b. Let $(\alpha D + \beta D' + \gamma)^n$ be a factor of $F(D, D')$ and $\alpha \neq 0$, then prove that **(07)**

$$u = e^{-\frac{\gamma x}{\alpha}} \sum_{s=1}^{n} x^{s-1}\, \phi_s(\beta x - \alpha y)$$

is a solution of $F(D, D')z = 0$.

Q-3 **Attempt all questions** **(14)**

a. Solve: $(D - 3D')^2(D + 3D')z = e^{3x+y}$. **(05)**

b. Solve: $(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x+y)$. **(05)**

c. Eliminate the arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$. **(04)**

OR

Q-3 **Attempt all questions** **(14)**

a. Reduce $x^2 r + y^2 t = 0$ to canonical form. **(05)**

b. Solve: $r + s - 2t = (2x+y)^{\frac{1}{2}}$. **(05)**

c. Solve: $\frac{\partial^4 z}{\partial x^4} - 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = 0.$ **(04)**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a. State Green's identities. **(02)**

b. What is equipotential surface? **(02)**

c. Write the Laplace equation in spherical co-ordinate system. **(02)**

d. Wave equation is considered in the Dirichlet BVP. Determine whether the statement is True or False. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Show that the solution of three dimensional wave equation can be put in the form $e^{\pm i(lx+my+nz+kct)}$, where l, m, n, k are constants with $k^2 = l^2 + m^2 + n^2$. **(07)**

b. Solve the following boundary value problem in the half-plane $y > 0$, described by **(07)**

PDE: $u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty, y > 0$

BCs : $u(x,0) = f(x), \quad -\infty < x < \infty$,

u is bounded as $y \to \infty$, u and $\frac{\partial u}{\partial x}$ vanish as $|x| \to \infty$.

OR

Q-5 **Attempt all questions** **(14)**

a. Solve interior Dirichlet problem for a function $u = u(r,\theta)$ for circle and show that solution is of the form $u(r,\theta) = \sum_{n=0}^{\infty} r^n (A_n \cos n\theta + B_n \sin n\theta)$, with A_n, B_n are constants. **(07)**

b. Show that if the three dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ is transform to cylindrical coordinates (r,θ,z) defined by $x = r\cos\theta, y = r\sin\theta$, is takes the form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0.$ **(07)**

Q-6 **Attempt all questions** **(14)**

a. Let D be a bounded domain in R^2, bounded by a smooth closed curve B. Let $\{u_n\}$ be a sequence of functions, each of which is continuous on $\overline{D}$ and harmonic in D. If $\{u_n\}$ converges uniformly on B, then prove that $\{u_n\}$ converges on $\overline{D}$ to limit function which is continuous on $\overline{D}$ and harmonic in D. **(05)**

b. Show that the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form an equipotential family of surfaces, and find the general form of the corresponding potential function. **(05)**

c. Using the method of separation of variables, solve **(04)**

$$u_x + 2u_y = 0, u(0,y) = 4e^{-2y}.$$

OR

Q-6 **Attempt all questions** **(14)**

a. Suppose that $u(x,y)$ is harmonic in bounded D and continuous in $\overline{D} = D \cup B$. Then prove that u attains its maximum on the boundary B of D. **(05)**

b. Solve: $x^2 r - y^2 t + px - qy = \log x$. **(05)**

c. Find the Green's function for the following boundary value problem **(04)**

$$y''(x) = f(x), y(0) = 0, y(1) = 0.$$